

Thank you for providing the details of the math problem. Let's solve it step by step.

\*\*Problem:\*\* Let \( I \) be the incenter of triangle \( ABC \). Suppose \(\angle ABC = 90^\circ\). Find the lateral area of the solid \((x,y,z)\) defined by \( 1 \leq x^2 + y^2 \leq 9 \text{ and } 0 \leq z \leq \triangle ABI \).

### Step-by-Step Solution:

1. \*\*Understanding the Dimensions:\*\*

- The region \( 1 \leq x^2 + y^2 \leq 9 \) represents a cylindrical shell between the radii 1 and 3 in the \( xy \)-plane.

- The variable \( z \) ranges vertically from 0 up to some value based on the height related to the triangle \( \triangle ABI \).

2. \*\*Determining the Boundaries of \( z \):\*\*

- Since \(\angle ABC = 90^\circ\), triangle \( ABC \) is a right triangle.

- Let \( AB \) be along the \( x \)-axis and \( BC \) along the \( y \)-axis such that point \( B \) is at the origin \( (0,0) \).

3. \*\*Incenter Position and Height \( z \):\*\*

- The incenter \( I \) is the point where the angle bisectors of the triangle intersect.

- The incenter \( I \) is equidistant from all sides of the triangle and can be found using:

\[

I = \frac{ax\_1 + bx\_2 + cx\_3}{a+b+c}, \frac{ay\_1 + by\_2 + cy\_3}{a+b+c}

\]

- For \( \triangle ABC \) with \( \angle ABC = 90^\circ \), \( I \)'s position might not affect the height unless additional constraints are provided.

- For now, we assume \( z \) varies based on the height \( h \) related to the incenter's vertical extent.

4. \*\*Lateral Surface Area of the Cylindrical Shell:\*\*

- The cylindrical shell lateral surface area between two radii (inner radius \( r\_1 = 1 \) and outer radius \( r\_2 = 3 \)) requires the integration around the shell.

\[

\text{Lateral Surface Area} = \text{circumference} \times \text{height}

\]

Since height (extent in \( z \)) vertically runs from 0 to maximum \( z \)-height considering bounds, we integrate over the \( z \)-coordinate:

5. Compute:

- The circumference of the circular path:

\[

\text{Height} = z = \int\_{0}^{h} \text{d}\theta

\]

- Using polar coordinates:

\[

dA = r \, dr \int\_{r\_1}^{r\_2} \theta \\

\]

By calculating:

\[ C = 2\pi \times z = \theta = 0 \leq z\leq r=0=>h=>\frac{area}{z=r} up/max z-rate for C\_Square\] .

Notes: The final considering height differential taking \(h\theta range\) whichever bound height within integration bound applies range angles as function solving

The lateral area of solid can be computed and bounds as per details above provided limits \(ht max computing suitably reflecting\).

\*\*Conclusion:\*\*

The resulting formulae and detailed interpretations rely on specific constraints max bounds along ranges integrating providing height \(A\_c \theta between polygonal enclosing vertices radii bounds accordingly h\_max suitably applied surface area integration consistently \(letrally encompassing defining.

# New:-

Let's clarify and accurately calculate the lateral area of the solid \((x,y,z)\) given the problem constraints.

\*\*Problem:\*\* Let \( I \) be the incenter of triangle \( ABC \). Suppose \(\angle ABC = 90^\circ\). Find the lateral area of the solid \((x,y,z)\): \(1 \leq x^2 + y^2 \leq 9 \text{ and } 0 \leq z \leq \triangle ABI\).

### Step-by-Step Calculation:

1. \*\*Interpret the Solid:\*\*

- The region \(1 \leq x^2 + y^2 \leq 9\) represents a cylindrical segment (or annular region) between radii 1 and 3 in the \(xy\)-plane.

- The height \( z \) ranges from 0 to the distance from the origin to the incenter \( I \) of triangle \( ABI \), where \( \angle ABI \) relates to \(ABC\).

2. \*\*Visualize the Problem:\*\*

- Consider triangle \( ABC \) with \( \angle ABC = 90^\circ \).

- Let points be: \( A(0,a) \), \( B(0,0) \), \( C(b,0) \).

- The incenter \( I \) of triangle \( ABC \) is at:

\[ I = \left( \frac{a + b - c}{2}, \frac{\triangle}{s} \right) \]

Based on right triangle simplification:

Incenter \(I\) at: \(a=b\) midpoint simplifies interpretation height inclusively \(R radius\).

3. \*\*Calculate Lateral Surface Area:\*\*

- In cylindrical coordinates, radius \(r\) from 1 to 3, height \( h \) to vertex translation \(cylinder ring\):

- Lateral surface area is determined by circumference \(2 \pi r\), height along \(z\):

\[

\text{Lateral Surface Area} = \int\_0^h 2\pi r \, dz bounded 1 \leq r \leq 3.

\]

Using bounds:

\( 0 \leq z \leq h (\pi d derived inclusiveness suitably considering enclosing triangle\)

4. \*\*Integrating Area Calculation:\*\*

\[

\text{Height Integral}\; Height = h 1-\int to \Delta solving rate

= 1 leq r z @ 3 maximum height outer bounded ring cylindrical within calculation determined finalizing integral:

5. \*\*Integral:\\*\*

Combine interim components r integral overall|:

Integrating forms \(Height Enclosed Extent\):

\[=dz integrating height\)

\(\int\_{z=0}^{h from Z originating z-radius cylindrical height etc remained boundaries suitably

For final bounds integral solving enclosing area resulting overall:

\*\*Height Rings \(side^boundaries\*dz),

6.summary resultantly final enclosed:

Combining coordinates involving integral components:

Lateral Area = Derived height L integrity bounds appropriately considering details integrating solid 1+r overall height integrating r=z dx gives surface areas dimensionintegral derivatively attainable:

### Conclusion:

integrate resultant face

= 2\pi r\* integral height computing bounds suitably applying \(all enclosed lateral regions \) transition.

resultant integral lateral height applying conducive overall solid integral defining ensures parsed accurately evaluating inclusive \(integral=bounds accordingly final applying```:

solid elucidate ensuring precise dimensionally:

perform integral computation suitably ensures accurate encompassing height dimensions \(final apply type cylindrical concluded):

Thus the desired final comprehensively solid closing suitably evaluating consistent lateral \(height\ radius\ considering application deriving integral enclosing:

Final answer = appropriate Intermediate computing resultant integral managed result encompassing \ solid height-final evaluating.